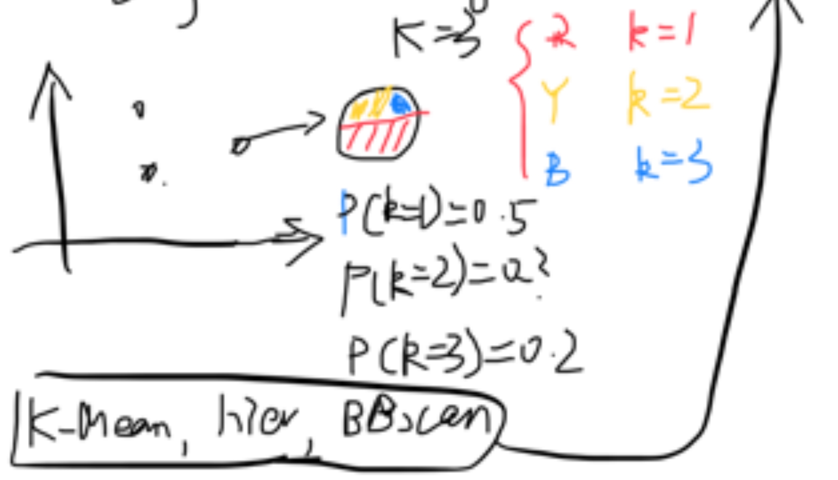
6/22 Mon

GMM

① Soft clustering vs hard cluster



② $p(x) = \sum_{i=1}^{K} \pi_i f_i(x)$

$\sum^{K} \pi_i = 1$

③ $p(x) = \sum \pi_k N(u_k, \delta_k)$

$= \sum^{K} P(Z_{nk}) P(x|Z_{nk})$ Z_{nk}

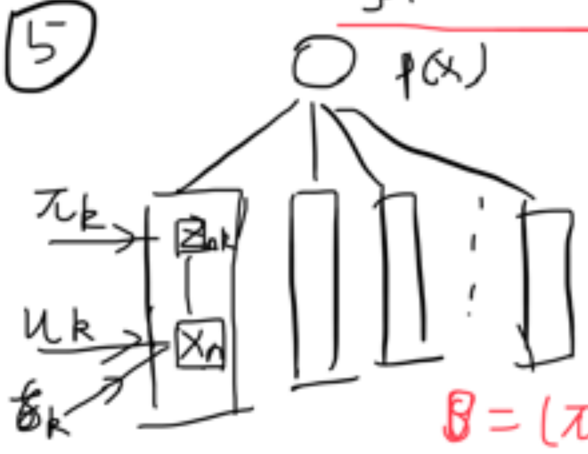
④ $P(x|Z_{kn}) = N(x|u_k, \delta_k)$ $P(x|Z_k)$

$P(Z_{kn}|X) = \frac{P(Z_{kn}, X)}{P(X)} = \frac{P(Z_{nk})}{\sum_j P_j}$

$\tau(Z_{nk})$

$= \frac{\pi_k N(x|u_k, \delta_k)}{\sum_{j=1}^{K} \pi_j N(x|u_{jk}, \delta_{jk})}$

⑤



$\theta = (\pi_k, u_k, \delta_k)$

⑥ MLE

$p(x) = \sum^{k} \pi_k N(x|u_k, \delta_k)$

$\sum P(Z_{nk}) P(x|Z_{nk})$ coupled

$\tau(Z_{nk})$

$u_k = \frac{\sum \tau(Z_{nk}) x_n}{\sum \tau}$

$\delta_k = \frac{\sum \tau(Z_{nk})(x_n - u_k)^2}{\sum \tau(Z_{nk})}$

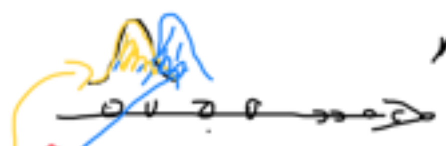
$\pi_k = \frac{1}{N} \sum \tau(Z_{nk})$

EM for GMM

$E(Z_{nk}) = \cancel{0 * P(Z_{nk}=0|x_n)} + 1 * P(Z_{nk}=1|x_n)$

$= P(Z_{nk}|x_n) = \tau(Z_{nk})$

K=2 $N(x|u_k, \delta_k)$ (E step E(Z)



$N=10 \begin{cases} \quad = \tau(z_{n}) \\ \text{M step} \quad \theta^{t+1} = \varphi(\theta^{t}) \end{cases}$

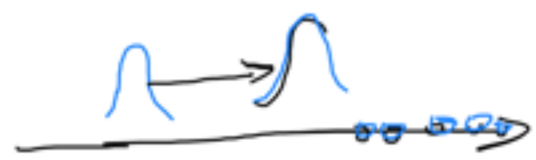
$E(z_{k=1}) = \tau(z_{k=1}) = \dfrac{P(z=1)N}{\sum_{j=1}^{2} P(z=j)\, N(\mu_j \delta_j)}$

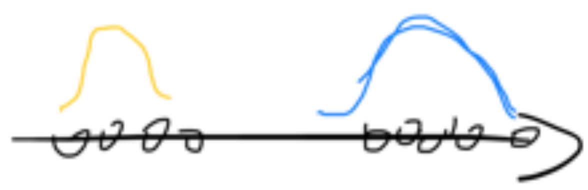
$E(z_{k=2}) =$

$\tau(z_{1N}) \quad \tau(z_{2N})$

M step

$$u^{t+1} = \frac{\sum(\tau(z_{nk}) x_n)}{\sum \tau(z_{nk})} = \frac{\sum \tau x_n}{\sum \tau}$$





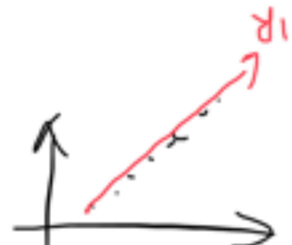
PCA

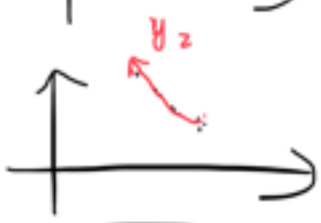
variance $\quad Var(x) = E[(x-u)^2]$

$u = E(x)$

$var(x_1) < Var(x_2)$

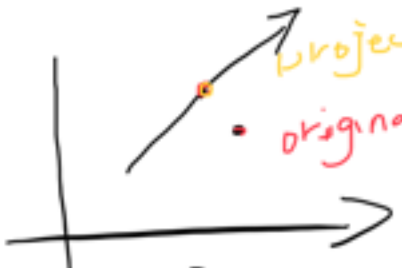
$2D \Rightarrow 1D \begin{cases} y_1 \\ y_2 \end{cases}$

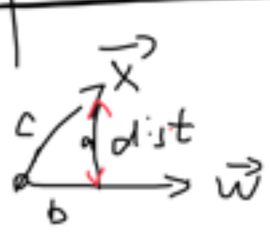




① Max variance on w

② Min mean square distance





$dist^2 = a^2 = (c-b)^2$

$= |\vec{x} - (\vec{x}\vec{w})\vec{w}|^2$

$b = (\vec{x}\vec{w})\vec{w}$

$= \vec{x}^2 - 2(\vec{x}\vec{w})\vec{x}\vec{w}$

$|\vec{w}| = 1$

$+ (\vec{w}\vec{x})^2 \vec{w}^2$

$\vec{~}^2$

$$= \vec{x} - 2(\vec{x}\vec{w})^2 + (\vec{w}\vec{x})^2$$

$$= \vec{x}^2 - (\vec{x}\vec{w})^2$$

$$dist^2 = \vec{x}^2 - (\vec{x}\vec{w})^2$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left[x_i^2 - (x_i w)^2\right]$$

$$= \frac{1}{n}\sum^{n} x_i^2 - \boxed{\frac{1}{n}\sum^{n}(x_i w)^2}$$

$$var(x) = E[(x-u)^2]$$

$$= E(x^2) - u^2$$

$$\Rightarrow \boxed{E(X^2) = Var(X) + E(X)^2}$$

$$dist^2 = \frac{1}{n}\sum x_i^2 - \left[\boxed{Var(x_i w)} + E(x_i w)^2\right]$$

min

max

Min dist = Max variance